Gravitational Radiation by Cosmic Strings in a Junction

R. Brandenberger*

Physics Department, McGill University, Montreal, H3A 2T8, Canada., and Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-4, Beijing 100049, P.R. China

H. Firouzjahi†

School of Physics, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran.

J. Karouby‡

Physics Department, McGill University, Montreal, H3A 2T8, Canada.

S. Khosravi§

Physics Department, Faculty of Science, Tarbiat Mo'alem University, Tehran, Iran, and School of Astronomy, Institute for Research in Fundamental Sciences(IPM), Tehran, Iran.

Abstract

The formalism for computing the gravitational power radiation from excitations on cosmic strings forming a junction is presented and applied to the simple case of co-planar strings at a junction when the excitations are generated along one string leg. The effects of polarization of the excitations and of the back-reaction of the gravitational radiation on the small scale structure of the strings are studied.

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PACS numbers:

*Electronic address: rhb@physics.mcgill.ca

†Electronic address: firouz@ipm.ir

‡Electronic address: karoubyj@physics.mcgill.ca

§Electronic address: khosravi@ipm.ir

I. INTRODUCTION

In models of brane inflation cosmic strings are produced (for a review see [1]). This has led to a revival of interest in cosmic strings (see e.g. [2, 3, 4]). Cosmic strings forming in the context of brane models can take the form of Fundamental strings (F-strings), D1-branes (D-strings) or their bound states, ((p,q) strings). A (p,q) string is a bound state of p F-strings and q D-strings. Networks of stringy cosmic strings which can involve strings with different values of p and q have features unlike those of simple gauge theory strings. Unlike U(1) gauge theory cosmic strings which inter-commute when they intersect, in the case of cosmic (p,q) strings there are conservation laws which prevent the inter-commutation of strings with different values of p and q. Instead, a string junction can be formed. For example, a p string and a q string can join at a junction to form a (p,q) string. The construction of cosmic strings with junctions and its cosmological implications were studied in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

Gravitational wave (GW) emission from loops and cusps of cosmic strings has been studied (for a comprehensive review see [17, 18] and for more recent analyses see [19, 20]). A straight infinite string does not emit GW. This is because to emit GW, as we shall explicitly see in the next section, both left-movers and right-movers should be present on the string world sheet. In a network of cosmic strings, it is quite natural to expect that wiggles of different wavelengths are generated on the world sheet of an infinite string. These wiggles, for example, are left over from times when the correlation length of the string network was much smaller, or are remnants of string inter-commutations which took place in the past. These wiggles cause the GW emission from long strings and can smooth out the wiggles of the string world sheet. GW emission from wiggles on a straight string were studied in [21, 22, 23]. In particular, in [23] left-moving and right-moving wave-trains of different wavelengths and amplitudes on an infinite string were considered. It was shown that when the wavelengths and the amplitudes of the wave-trains are comparable, the GW emission is mainly from lower harmonics and is proportional to the frequency of the wave-trains. This indicates that excitations of higher frequency die out faster than excitations of shorter frequencies. On the other hand, when the wavelengths and amplitudes of the wave-trains are much different, then GW emission is exponentially suppressed.

As mentioned above, the formation of junctions is a generic feature of networks of cosmic

superstrings. With this motivation, in this paper, we consider gravitational radiation from strings at a junction. As we shall see, the presence of the junction leads to mixing of left and right-moving excitations on the string which is the necessary criterium for the emission of GW. In Section 2, we present the setup of our study. In Section 3 we study three examples. The first example is GW emission from a semi-infinite string attached to a rigid wall. The second example corresponds to GW emission from a stationary junction. The third example concerns GW emission from a non-stationary junction. As we shall see, the expressions for the gravitational wave power radiated has a similar form in all three examples. We discuss our results and summarize our conclusions in Section 4.

II. THE SETUP

Our setup consists of semi-infinite strings forming a stationary junction. The formalism in this section is valid for any number of semi-infinite strings meeting at a junction. However, to be specific, in our study we shall focus on the simple example where three semi-infinite strings form a stationary junction.

The world-sheet of each string is described by a temporal coordinate τ and a string length parameter σ. The induced metric $\gamma_{i\,ab}$ on each string is given by

$$\gamma_{i\,ab} = g_{\mu\nu}\,\partial_a\,X_i^\mu \partial_b\,X_i^\nu\,. \tag{1}$$

Here and in the following, we reserve $\{a,b\} = \{\tau,\sigma\}$ for the string world-sheet indices while Greek indices represent the four-dimensional space-time coordinates. Furthermore, X_i^μ stands for the position of the i-th string in four space-time dimensions.

We impose the conformal temporal gauge on the string world-sheet for which $X_i^0 = t = \tau$ and $\gamma_{i\,0\sigma} = 0$. This is equivalent to

$$\dot{\mathbf{x}}_i\,.\,\mathbf{x}'_i = 0 \quad , \quad \dot{\mathbf{x}}_i^2 + \mathbf{x}_i'^2 = 1\,. \tag{2}$$

Here an overdot and a prime denote derivatives with respect to t and σ, respectively, while $\mathbf{x}_i$ represent the spatial components of the i-th string.

For the components of the induced metric on the string world sheet we obtain

$$\gamma_{i\,00} = 1 - \dot{\mathbf{x}}_i^2 \quad , \quad \gamma_{i\,\sigma\sigma} = -\mathbf{x}_i'^2 = -\gamma_{i\,00}\,. \tag{3}$$

We start with the following action

$$S = -\sum_i \mu_i \int d\,t\,d\,\sigma \sqrt{-|\gamma_i|}\,\theta(s_i(t) - \sigma) \tag{4}$$

where $|\gamma_i|$ is the determinant of the world sheet metric of the i-th string. We are using the convention that the position of the junction on the i-th string is given by $s_i(t)$. It is assumed that σ is increasing towards the junction. We can impose a lower cutoff on σ, which would correspond to the physical length of the string under consideration. The equation of motion for $s_i(t)$ and the conditions for junction formation have been studied in [6, 7, 8].

The energy-momentum tensor for the action given by Eq. (4) is obtained by varying the action with respect to the background metric $g_{\mu\nu}$, with the result

$$\delta_{g_{\mu\nu}} S = -\frac{1}{2}\sum_i \mu_i \int d\,t\,d\,\sigma \sqrt{-|\gamma_i|}\,\gamma^{ab}\partial_a\,X_i^\mu \partial_b\,X_i^\nu\,\theta(s_i(t) - \sigma)\,\delta g_{\mu\nu} \equiv -\frac{1}{2}\int d^4x\,T^{\mu\nu}\,\delta g_{\mu\nu} \tag{5}$$

which gives

$$\begin{aligned} T^{\mu\nu}(x) &= \sum_i \mu_i \int d\,t\,d\,\sigma \sqrt{-|\gamma_i|}\,\gamma^{ab}\,X^\mu_{i,\,a}\,X^\nu_{i,\,b}\,\theta(s_i(t) - \sigma)\,\delta^{(4)}(x - X_i) \\ &= \sum_i \mu_i \int d\,t\,d\,\sigma(\dot{X}^\mu\dot{X}^\nu - X'^\mu X'^\nu)\,\theta(s_i(t) - \sigma)\,\delta^{(4)}(x - X_i)\,. \end{aligned} \tag{6}$$

Having obtained the energy-momentum tensor, we can use the standard formalism for calculating GW emission from a source [24] .The derivation in [24] is for a source which is localized in space. To justify the application of the formalism to the case of a long string, we can imagine considering first short wave-trains on the string, in which case the formalism of [24] applies as it was initially derived, and then taking the limit in which the length of the wave trains increases. This limit does not lead to any problems when applying for formalism . According to this formalism, the power emitted in direction $\mathbf{k}$ per solid angle Ω, integrating over the frequencies ω of the emitted waves, is given by

$$\frac{dE}{d\Omega} = 2G\int_0^\infty d\omega\omega^2\left[T^{\lambda\nu*}(k)\,T_{\lambda\nu}(k) - \frac{1}{2}|T^\lambda_\lambda(k)|^2\right]\,, \tag{7}$$

where G is Newton's gravitational constant and $T_{\lambda\nu}(k)$ is the Fourier transform of $T_{\lambda\nu}(t,\mathbf{x})$

$$T_{\mu\nu}(k) = \frac{1}{2\pi}\int d^4x\,T_{\mu\nu}(x)\,e^{ik.x}\,. \tag{8}$$

In conformal temporal gauge the solution of the string equations of motion

$$\ddot{X}^\mu - X''^\mu = 0 \tag{9}$$

can be represented by the combination of left-moving and right-moving modes:

$$X_i^\mu = \frac{1}{2}\left(a_i^\mu(v) + b_i^\mu(u)\right) \quad , \quad {a_i'}^2 = {b_i'}^2 = 0\,. \tag{10}$$

where $v = \sigma + t$ and $u = \sigma - t$ are the light-cone coordinates.

Since we need the components of the energy-momentum tensor in Fourier space, it is useful to replace the θ function by its Fourier representation which is

$$\theta(x) = \frac{1}{2\pi\, i}\int_{-\infty}^{\infty} \frac{d\ell\, e^{i\ell x}}{\ell - i\varepsilon} \quad , \quad \varepsilon \to 0^+\,. \tag{11}$$

Inserting this into Eq. (6), we find

$$T^{\mu\nu}(k) = \sum_j \frac{\mu_i}{8\pi}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{d\ell}{\ell - i\varepsilon} \quad \int du dv (a_j'^\mu b_j'^\nu + a_j'^\nu b_j'^\mu)$$
$$\times \exp\left[i\ell s_j(u,v) - \frac{i\ell}{2}(u+v) + \frac{i}{2}k.(a_j + b_j)\right]. \tag{12}$$

In the first two examples in the following section, we consider cases when the junction remains stationary, corresponding to $s_i = 0$. In this case, one obtains

$$T^{\mu\nu}(k) = \sum_j \frac{\mu_j}{8\pi}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{d\ell}{\ell - i\varepsilon}\left(A_j^\mu(k,\ell)B_j^\nu(k,\ell) + A_j^\nu(k,\ell)B_j^\mu(k,\ell)\right) \tag{13}$$

where

$$A_j^\mu(k,\ell) \equiv \int_{-L/2}^{L/2} dv\, a_j'^\mu(v)\, \exp\left[ik.a_j(v)/2 - i\ell v/2\right]$$
$$B_j^\mu(k,\ell) \equiv \int_{-L/2}^{L/2} du\, b_j'^\mu(u)\, \exp\left[ik.b_j(u)/2 - i\ell u/2\right]\,, \tag{14}$$

where L is the physical length of the string being considered (since the GW emission comes from regions where the wave trains are non-vanishing, effectively L can be taken as the length on the string which corresponds to the region where the wave-trains are localized), and we assumed the mid point of the string is at world sheet coordinates $u = v = 0$.

To calculate $A_i^\mu(k,\ell)$ and $B_i^\mu(k,\ell)$ we follow the formalism of [23]. We assume that on each string there are left-moving and right-moving wave-trains of lengths $L_i = N_a^i \lambda_a^i$ and $L_i = N_b^i \lambda_b^i$ for integers N_a^i and N_b^i, where $\lambda_{a(b)}^i = 2\pi/\kappa_{a(b)}^i$ are the wavelengths of the left(right)-moving wave-trains on each string.

We are interested in GW emission from the excitations of the strings and neglect the contributions of the straight parts of the strings to $T^{\mu\nu}$. To establish our notation, the

contributions to a quantity Q from the string excitations are denoted by δQ. For example, $a_i^\mu \to a_i^\mu + \delta a_i^\mu$ and so on. Discarding the contributions from the straight parts of the strings (which do not contribute to gravitational radiation), one obtains for the fluctuating part of $T^{\mu\nu}$:

$$\delta T^{\mu\nu}(k) = \sum_j \frac{\mu_j}{8\pi} \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{d\ell}{\ell - i\varepsilon} (\delta A_j^\mu \delta B_j^\nu + \delta A_j^\nu \delta B_j^\mu)\,, \tag{15}$$

where

$$\delta A_j^\mu(k,\ell) = \int_{-L/2}^{L/2} dv\, e^{iv\hat{K}_+^j/2} \left(\delta a_j'^\mu + \frac{i}{2} a_j'^\mu k.\delta a_j \right) \tag{16}$$

and $\delta B^\mu(k,\ell)$ is given by a similar expression. Here

$$\hat{K}_+^j \equiv K_+^j - \ell \quad , \quad K_+^j \equiv k.a_j' \tag{17}$$

with $\hat{K}_-$ and K_- defined similarly for the right-movers.

For the case where $s_i \neq 0$, such as in the third example in the next section, we obtain

$$\begin{aligned} \delta T^{\mu\nu}(k) &= \sum_j \frac{\mu_j}{8\pi} \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{d\ell}{\ell - i\varepsilon} \{ \ (\delta A_j^\mu \delta B_j^\nu + \delta A_j^\nu \delta B_j^\mu) \\ + \ i\ell &\int du dv s_j e^{i\hat{K}_+^j v/2} e^{i\hat{K}_-^j u/2} \left[(a_j'^\mu \delta b_j'^\nu + b_j'^\nu \delta a_j'^\mu + \mu \leftrightarrow \nu) + \frac{i}{2} k.(\delta a_j + \delta b_j)(a_j'^\mu b_j'^\nu + a_j'^\nu b_j'^\mu) \right] \\ - \ \frac{\ell^2}{2} &\int du\, dv\, s_j^2 e^{i\hat{K}_+^j v/2} e^{i\hat{K}_-^j u/2} (a_j'^\mu b_j'^\nu + a_j'^\nu b_j'^\mu) \ \} \end{aligned} \tag{18}$$

III. EXAMPLES

In this section we employ the formalism presented in the last section to calculate GW emission for different examples.

A. A semi-infinite string attached to the wall

The first example we would like to consider is gravitational radiation from a semi-infinite string attached to a rigid wall. An incoming perturbation is coming from infinity, hits the wall and gets reflected. This creates wave-trains of both left-mover and right-mover on the string. This problem is in spirit very similar to the problem of two left-moving and right-moving wave-trains propagating on an infinite string studied by Siemens and Olum [23].

The non-fluctuating string configuration is given by

$$a'^\mu = (1, \mathbf{e}) \quad , \quad b'^\mu = (-1, \mathbf{e}) \tag{19}$$

where the unit vector $\mathbf{e}$ represents the orientation of the string. We could simply take the vector $\mathbf{e}$ to be along the z axis. However, in order to establish a formalism which can also be applied to the next examples involving strings oriented in different directions, we keep the vector $\mathbf{e}$ unspecified. The perturbations on the string are given by

$$\delta a'^\mu = \epsilon_a \mathbf{f}\ \cos(\kappa_a v) \quad , \quad \delta b'^\mu = \epsilon_b \mathbf{f}\ \cos(\kappa_a u) \tag{20}$$

where $\epsilon_{a(b)}$ are small numbers controlling the amplitude of the perturbations, $\kappa_{a(b)}$ are the frequencies of the left(right)-moving perturbations and $\mathbf{f}$ is a unit vector indicating the polarization of the perturbations with $\mathbf{e}.\mathbf{f} = 0$. In this example we know that $\epsilon_a = \epsilon_b$ and $\kappa_a = \kappa_b$. However, in order to keep the formalism general we have not made these identifications.

Calculating δA^μ, one obtains

$$\delta \vec{A} = \delta A^0 \left(\mathbf{e} - \frac{\hat{K}_+}{\mathbf{k}.\mathbf{f}} \mathbf{f} \right) \quad , \quad \delta A^0 = -4\,(-1)^{N_a} \epsilon_a\, \mathbf{k}.\mathbf{f}\ \frac{\sin(L\hat{K}_+/4)}{\hat{K}_+^2 - 4\kappa_a^2} \tag{21}$$

with a similar expressions for δB^μ with $\hat{K}_+$ replaced by $\hat{K}_-$.

To calculate $\delta T^{\mu\nu}(k)$, we need to plug $\delta A^\mu(k,\ell)$ and $\delta B^\mu(k,\ell)$ into Eq. (15) and integrate over ℓ. There are five poles at

$$\ell_1 = i\varepsilon \quad , \quad \ell_{2,3} = K_+ \pm 2\kappa_a \quad , \quad \ell_{4,5} = K_- \pm 2\kappa_a \,. \tag{22}$$

One can easily check that only the residue at ℓ_1 gives a non-zero contribution to $\delta T^{\mu\nu}(k)$ and the other residues vanish. For example, calculating the residue at $\ell = \ell_2$, one obtains that $\delta T^{\mu\nu}(k) \propto \sin(L\kappa_a/2) = \sin(N_a \pi) = 0$.

Calculating the residue at $\ell = \ell_1$, one obtains

$$\delta T^{\mu\nu}(k) = \frac{\mu_1}{8\pi} \left[\delta A^\mu(k,0) \delta B^\nu(k,0) + \delta A^\nu(k,0) \delta B^\mu(k,0) \right] . \tag{23}$$

Noting that $\delta A^\mu(k,0)$ and $\delta B^\mu(k,0)$ are real, one obtains

$$\begin{aligned} \frac{dE}{d\Omega\, d\omega} &= \frac{G\mu_1^2}{16\pi^2} \omega^2 \delta A(k,0)^2 \delta B(k,0)^2 \\ &= \frac{16\, G\mu_1^2}{\pi^2} \omega^2 \epsilon_a^2 \epsilon_b^2\, K_+^2\, K_-^2\, \frac{\sin^2(LK_+/4)}{(K_+^2 - 4\kappa_a^2)^2}\, \frac{\sin^2(LK_-/4)}{(K_-^2 - 4\kappa_b^2)^2} \end{aligned} \tag{24}$$

One interesting result which emerges from the above is that the combination $\mathbf{k}.\mathbf{f}$ drops from the numerator and denominator of the above expression. This indicates that the GW power emission is independent of the polarization of the incoming waves. This feature will also show up in next example.

To exploit the symmetry of the problem, now we assume that the string is oriented along the z axis, so $\mathbf{k}.\mathbf{e} = \omega \cos\theta$, where θ is defined as the angle between the vector $\mathbf{k}$ and the orientation of string. On the other hand, calculating K_+ and K_-, we get

$$K_+ = \omega(1-\cos\theta) \quad , \quad K_- = -\omega(1+\cos\theta)\,. \tag{25}$$

As in [23], changing the coordinates from $(\omega, \cos\theta)$ to (K_+, K_-), and noting that $2\omega = K_+ - K_-$ and $d\omega\, d(\cos\theta) = dK_+ dK_-/2\omega$, one obtains

$$\frac{dE}{d\phi} = \frac{4\,G\mu_1^2}{\pi^2}\epsilon_a^2\epsilon_b^2 \int dK_+ dK_- (K_+ - K_-) K_+^2 K_-^2 \,\frac{\sin^2(LK_+/4)}{(K_+^2 - 4\kappa_a^2)^2}\,\frac{\sin^2(LK_-/4)}{(K_-^2 - 4\kappa_b^2)^2}\,. \tag{26}$$

Here ϕ is defined as the azimuthal angle around the string.

The above integral can be performed using the following approximations for large N_a [23]

$$\frac{\sin^2(N_a x^a/2)}{\sin^2(x^a/2)} = 2\kappa_a N_a \sum_{n=-\infty}^{\infty} \delta(K_+ - 2n\kappa_a) \tag{27}$$

where $x_a \equiv \lambda_a K_+/2$. A similar identity also holds for right-movers with $K_+ \to K_-$, $N_a \to N_b$ and $n \to m$. Using this identity, Eq. (26) yields

$$\frac{dE}{d\phi} = \frac{2G\mu_1^2}{\pi^2}\frac{N_a N_b}{\kappa_a\kappa_b}\sum_{m,n}\frac{n^2 m^2 (n\kappa_a - m\kappa_b)\sin^2(n\pi)\sin^2(m\pi)}{(n^2-1)^2(m^2-1)^2}\,. \tag{28}$$

Knowing that $K_+ \geq 0, K_- \leq 0$, one can see that only $n = -m = 1$ contribute in the summation in Eq. (28) and one obtains

$$\frac{dE}{d\phi} = \frac{G\mu_1^2}{8} N_a N_b \pi^2\, \epsilon_a^2\epsilon_b^2 \frac{\kappa_a + \kappa_b}{\kappa_a\kappa_b}\,. \tag{29}$$

We are interested in the power radiated per unit of length, dP/dl, which is obtained by dividing the above expression by the world-sheet volume of the string, $L_a L_b/2$. After integrating over the angle ϕ and noting that $\epsilon_a = \epsilon_b$ and $\kappa_a = \kappa_b$, we obtain

$$\frac{dP}{d\,l} = \frac{G\mu_1^2\pi}{16}\epsilon_a^4\kappa_a\,. \tag{30}$$

Note that our result is identical to the power radiated per unit of length obtained in [23] for an infinite string. This is not surprising since our semi-infinite string locally looks

identical to an infinite string. The only difference is at the junction with the wall - but since that point is not moving it does not contribute to the power of gravitational radiation. Thus, we expect that our result for a semi-infinite string agrees with that of [23] for an infinite string (there is a factor of 1/2 mistake in the original version of Eq. (72) of [23] which is corrected in the printed version. Taking that into account, our result here agrees with their Eq. (36)).

As in [23] the power radiation is dominated by lower harmonics. Also for $n = -m = 1$ we note that $\omega = 2\kappa_a$, so the frequency of the radiation is twice of the frequency of the incoming wave.

B. Strings at a junction

In this section we consider the problem of GW emission from strings at a junction. Three semi infinite strings form a stationary junction. There is an incoming right moving excitation on one string, say String 1. After the wave hits the junction, part of it is transferred to Strings 2 and 3, while part of the incoming wave is reflected along String 1 (see [7] for the details of the dynamics). Depending on the polarization of the incoming wave, the junction may stay stationary, corresponding to $\delta s_i(t) = 0$, or the junction may dislocate along the strings corresponding to $\delta s_i(t) \neq 0$.

To be specific, suppose that the strings are in the $x - y$ plane and their orientations are given by $\mathbf{e}_i = (\cos\theta_i, \sin\theta_i, 0)$ where θ_i is the angle of the i-th string with the x-axis. This gives

$$
\begin{aligned}
a_i'^{\mu} &\equiv (1, \mathbf{a}_i) = (1, \mathbf{e}_i) \\
b_i'^{\mu} &\equiv (-1, \mathbf{b}_i) = (-1, \mathbf{e}_i)\,,
\end{aligned}
\tag{31}
$$

where $\mathbf{a}_i(\mathbf{b}_i)$ indicates the spatial part of $a_i^{\mu}(b_i^{\mu})$. The relations $\sum \mu_i \cos\theta_i = \sum \mu_i \sin\theta_i = 0$ also must be satisfied if the junction is to be stationary (this is due to the force balance condition).

Now suppose there is a small incoming excitation on one string, say String 1, with

$$
\delta\mathbf{b}_1'(u) = \epsilon\, \mathbf{f}_1 \cos(\kappa u)\,,
\tag{32}
$$

and $\delta\mathbf{b}_2' = \delta\mathbf{b}_3' = 0$. Here $\epsilon \ll 1$ is a dimensionless parameter controlling the amplitude of the perturbation.

1. *The case* $s_i(t) = 0$

The simplest case is when $\mathbf{f}_1 = (0, 0, 1)$. This has the advantage that the junction does not dislocate on the strings: $\delta \dot{s}_i = 0$ [7] and we can use the formalism developed in the previous section. Following [7], one finds

$$\delta \mathbf{a}'_1 = \frac{\nu_1}{\mu} \epsilon \, \mathbf{f}_1 \cos(\kappa v) \quad , \quad \delta \mathbf{a}'_2 = \delta \mathbf{a}'_3 = \frac{2\mu_1}{\mu} \epsilon \, \mathbf{f}_1 \cos(\kappa v) \, , \tag{33}$$

where

$$\mu \equiv \mu_1 + \mu_2 + \mu_3 \quad , \quad \nu_1 \equiv \mu_2 + \mu_3 - \mu_1 \, . \tag{34}$$

With this initial condition, one sees that only δB_1^μ is non-zero. Furthermore, $\delta T^{\mu\nu}(k)$ is as given in Eq. (23), with δA_1^μ given as in Eq.(21). Thus, $dE/d\Omega \, d\omega$ has the same form as Eq. (24). We note that String 1 is in $x - y$ plane. But we can label the coordinate (or perform a coordinate transformation) such that String 1 is along the z- direction. To calculate the power radiated, we can simply use Eq. (29) with the identification $\epsilon_b = \epsilon$ and $\epsilon_a = \epsilon \nu_1 / \mu$, and as before, we note that ϕ is defined as the azimuthal angle around String 1. The power radiated per unit of length, using Eq. (29), therefore is

$$\frac{dP}{d\, l} = \frac{G \mu_1^2 \pi}{16} \frac{\nu_1^2}{\mu^2} \epsilon^4 \kappa \, . \tag{35}$$

One may wonder why this result has the same form as that in the previous example where a semi-infinite strings was attached to a rigid wall. The reason is that here the junction plays the role of the rigid wall. Indeed, the fact that the junction remains stationary makes this analogy more manifest. The effect of Strings 2 and 3 is to let parts of incoming waves be transferred to them. This has the effect that $\epsilon_a \neq \epsilon_b$.

2. *The case* $s_i(t) \neq 0$

Now we consider the case when the polarization of the incoming perturbation is in the xy-plane. Then s_i will also oscillate and the junction does not stay at a fixed position on each string [7]. As in [7] we assume

$$\mathbf{f}_i = (-\sin \theta_i, \cos \theta_i, 0) \tag{36}$$

and $\mathbf{e}_i.\mathbf{f}_i = 0$ for each string. As before, the incoming perturbation is along String 1 and is given by

$$\delta \mathbf{b}_1'(u) = \epsilon \mathbf{f}_1 \cos(\kappa u) \,. \tag{37}$$

Following [7] , one obtains

$$\delta \mathbf{a}_1' = -\frac{\nu_1}{\mu}\epsilon\, \mathbf{f}_1 \cos(\kappa v) \quad , \quad \delta \mathbf{a}_{2,3}' = \frac{2\mu_1}{\mu}\epsilon\, \mathbf{f}_{2,3} \cos(\kappa v) \,, \tag{38}$$

and

$$\delta s_1 = \frac{(\mu_2 - \mu_3)\nu_1}{\kappa\, \Delta}\epsilon \sin(\kappa t) \quad , \quad \delta s_2 = -\delta s_3 = -\frac{\mu_1 \nu_1}{\kappa\, \Delta}\epsilon \sin(\kappa t) \tag{39}$$

with ν_1 as given in Eq. (34) and $\Delta = \sqrt{\mu \nu_1 \nu_2 \nu_3}$, where $\nu_{2,3}$ are defined like ν_1 with the appropriate permutations.

With these forms of δs_i, one can show (see the **Appendix**) that both integrals containing the linear and the quadratic powers of $s_i(t)$ in $\delta T_{\mu\nu}(k)$ in Eq. (18) vanish.

One sees that $\delta T^{\mu\nu}$ is of the same form as Eq. (23) with δA_1^μ given as in Eq.(21) . Like in previous example, the effect of the junction is to make $\epsilon_a \neq \epsilon_b$. On the other hand, since the amplitude of δa_1^μ is the same as in the previous example where the polarization was along the z axis, we find that the power radiated is the same as before, given by Eq. (35).

The fact that the power of gravitational radiation when the polarization $\mathbf{f}_1$ is coplanar with the strings is the same as when the polarization is perpendicular to the plane of the strings may seem surprising. However, as mentioned in [7], one can consider the excitations on the strings as the propagation of massless particles. Using conservation of energy, one can check that the transmission and reflection indices for both polarizations are the same.

IV. DISCUSSION

In this paper, gravitational wave (GW) emission from strings at a stationary junction has been studied. We considered the simple case when three co-planar semi-infinite strings form a stationary junction. A purely left-moving wave, excited on one string, travels towards the junction. Part of it is reflected from the junction while the rest is transferred to other strings. The role of the junction therefore is to mix the left-moving and right-moving excitations which are necessary for GW emission.

We found that power of gravitational radiation is independent of the polarization of the incoming wave. Furthermore, its magnitude is proportional to the frequency of the incoming wave. This means that excitations of higher frequencies (shorter wavelengths) die out faster than excitations with lower frequencies (longer wavelengths).

In [23, 25, 26] the gravitational back-reaction effects on the small scale structure present on a long string are studied. Here we shall briefly apply their formalism to our case. An excitation of the form Eq. (32) leads to a change $\delta\mu$ in the mass per unit length of the string. This change takes the form

$$\delta\mu \sim \mu_1 \epsilon^2 \,. \tag{40}$$

The energy loss via gravitational radiation given by Eq. (30) or Eq. (35) leads to a decrease of this contribution:

$$\frac{d}{dt}(\delta\mu) = -\frac{dP}{dz} = -\frac{G\mu_1}{8\lambda}\pi^2 \frac{\nu_1^2}{\mu^2}\epsilon^2 \delta\mu \,. \tag{41}$$

This differential equation has the solution

$$\delta\mu \sim \exp\left(-t/\tau\right) , \tag{42}$$

where

$$\tau = \frac{8\lambda}{\pi^2 G\mu_1}\frac{\mu^2}{\epsilon^2 \nu_1^2} \,. \tag{43}$$

Excitations which survive until the present time t_0 are characterized by $\tau > t_0$. Taking $\epsilon \lesssim 1$, the minimum wavelength of excitations that can survive is thus approximately given by

$$\lambda_{min} \sim G\mu_1 \frac{\nu_1^2}{\mu^2}\, t_0 \,, \tag{44}$$

while on smaller scales the wiggles are exponentially suppressed.

In this analysis we have considered monochromatic wave in the form of Eq. (32). In [25] (see also [26]) the estimation of back-reaction was generalized to the case when higher harmonics of the initial Fourier modes on the long string are present and when not all the modes interact with all of the other modes. In this case, it was shown that the minimum wavelength is given by

$$\lambda_{min} = \left(G\mu_1\right)^n t_0 \tag{45}$$

where $n = 3/2, 5/2$ for radiation and matter dominated eras, respectively.

In this work we have considered GW emission from three co-planar strings forming a junction and assuming that excitations are originally generated on one string leg. It would be interesting to generalize this exercise to more realistic cases of an arbitrary number of strings in a junction when incoming waves of arbitrary frequencies and amplitudes are excited on each string. It would be interesting to see if the results of [23] hold, where it was shown that GW emission from left-moving and right-moving wave-trains on an infinite string is zero if the wave-trains have significantly different frequencies and amplitudes.

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Appendix : Higher powers of $s_i(t)$ in $\delta T_{\mu\nu}(k)$

In this Appendix we show that the integrals containing the powers of $s_i(t)$ in $\delta T_{\mu\nu}(k)$ in Eq. (18) vanish. To see this, consider the term quadratic in s_i^2 when the corresponding integral is

$$\begin{aligned}&\int_{-\infty}^{\infty} \frac{\ell^2\, d\ell}{\ell - i\varepsilon} \int du\, dv\, s_i^2 e^{i\hat{K}_+ v/2} e^{i\hat{K}_- u/2} \\ &\sim \int_{-\infty}^{\infty} \frac{\ell^2\, d\ell}{\ell - i\varepsilon} \int du\, dv\, e^{i\hat{K}_+ v/2} e^{i\hat{K}_- u/2} \sin^2\left(\frac{\kappa}{2}(v-u)\right) . \end{aligned} \tag{46}$$

The integral further simplifies to

$$\int_{-\infty}^{\infty} \frac{\ell^2\, d\ell}{\ell - i\varepsilon} \int du\, dv\, e^{i\hat{K}_+ v/2} e^{i\hat{K}_- u/2} (1 - \cos\kappa u\, \cos\kappa v - \sin\kappa u\, \sin\kappa v) . \tag{47}$$

The first term in the above bracket gives

$$\int_{-\infty}^{\infty} \frac{\ell^2\, d\ell}{\ell - i\varepsilon} \int du\, dv\, e^{i\hat{K}_+ v/2} e^{i\hat{K}_- u/2}$$
$$\sim \int_{-\infty}^{\infty} \frac{\ell^2\, d\ell}{\ell - i\varepsilon} \frac{\sin(\hat{K}_+ L/4)}{\hat{K}_+} \frac{\sin(\hat{K}_- L/4)}{\hat{K}_-} . \tag{48}$$

This integral has three poles at $\ell = i\varepsilon$, $\hat{K}_+ = 0$ and $\hat{K}_- = 0$. However, the residues at all three poles vanish. Performing the integrals for the other two terms in the bracket in Eq. (47), one can check that the residues at the poles vanish.

In conclusion, the integral in Eq. (47) and correspondingly the integral in Eq. (18) containing terms of second power in $s_i(t)$ vanish. Following the same strategy, one can check that the integral in Eq. (18) containing terms of linear power in $s_i(t)$ also vanishes.

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